TEM Properties of Shielded Homogeneous Multiconductor Transmission Lines with PEC and PMC Walls

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Abstract— **This paper describes a very fast computational technique for the solution of two-dimensional static boundary value problems within a homogeneous rectangular enclosure with any combination of perfect magnetic and electric conducting walls. A fast convergent series representation of the pertinent Green's function combined with a least-square MoM and point-matching technique leads to a fast and reliable numerical solution. Characteristic impedances of single and coupled shielded metallic rods are computed as an example application of the present method. The results are compared with those of the finite-element method with excellent agreement and much less CPU time.**

I. Introduction

AN IMPORTANT application for the solution of two-dimensional static boundary value problems is to evaluate the TEM properties of single or multiple conductor transmission lines having a circular, rectangular, or elliptical inner conductor located inside a rectangular shield, inside a trough, or between infinite parallel planes. Here "TEM properties" means the self and mutual capacitances per unit length, even and odd mode characteristic impedances, and the field pattern. This class of transmission line problems have been the subject of numerous investigations [1]–[8]. This is mainly due to their wide applications in the design of combline and interdigital bandpass filters and directional couplers. Recently a TEM approximation has been successfully applied to the evaluation of aperture and loop couplings within multiple coupled coaxial cavity filters [9, 10]. In this context one needs to calculate the characteristic impedance and the field pattern of a circular-rectangular coaxial line with a combination of PEC and PMC walls. Some of these combinations are depicted in Fig. 1.

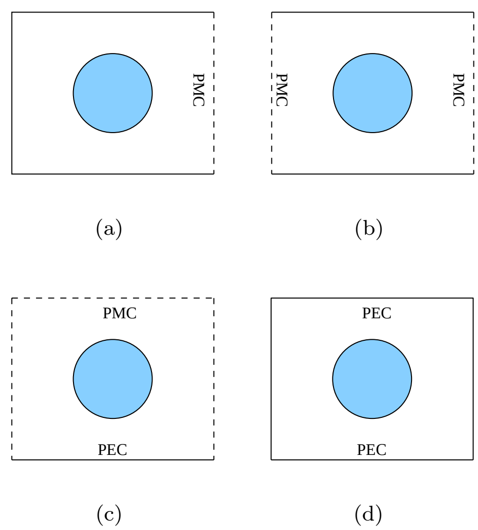

Fig. 1. Rectangular coaxial lines with PEC or PMC walls

Different approaches have been adopted for treatment of the above transmission line problems. A numerical inversion of the Schwartz-Christoffel transformation was employed by Costamagna *et. al.* [7, 8] to find the characteristic impedance of slab lines and concentric coaxial transmission lines. Levy [3] used an analytical-numerical method for coupled slab lines and the authors employed a modified Schwartz-Christoffel transformation for a rectangular-coaxial line [10]. Conformal mapping approach is usually impractical for asymmetrical or eccentric coaxial structures.

Variational technique as well as 2D integral equation method and MoM have also been applied to the above transmission line problems in different ways

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[1, 2, 5, 6]. Cristal [2] followed an integral equation approach using the 2D Green's function of the unbounded space in which the entire boundary of the inner and outer conductors are to be discretized. Stracca *et. al.* [5] used a direct summation of the side-wall images to calculate the Green's function which is cumbersome particularly when the inner conductor is very close to the walls.

In this article a method of moments with point matching and least square solution is introduced in which the required Green's function is expressed in terms of the so-called *theta functions* in complex plane [11]. An exponentially convergent series representation for the ϑ-function results in a very fast, robust, and accurate numerical method which can be applied to any transmission line structure with rectangular shield and any number of arbitrary shaped inner conductors. Numerical results are compared with those of the finite-element method with excellent agreement.

II. Formulation

A. Matrix Equation and Point Matching

Characteristic impedance of the TEM mode in a lossless uniform transmission line is related to its static capacitance per unit length through the following equation:

$$Z_c = \frac{1}{vC} = \frac{\sqrt{\mu\epsilon}}{C} \qquad (1)$$

where the capacitance C is equal to the total surface charge Q per unit length on one of the conductors when a potential difference of $V = 1$ is applied between the two conductors. As shown in Fig. 2 the inner cylinder[1] is replaced by a number N of line charges q_i placed on a circle of diameter $d' < d$ *inside* the conductor surface. Upon enforcement a constant potential boundary condition at M observation points on the surface of the inner conductor the following matrix equation is obtained:

$$\sum_{i=1}^{N} G_{ji}\, q_i = 1 \qquad j = 1, 2, \ldots, M \qquad (2)$$

where G_{ji} are the values of the pertinent Green's function. The above approach eliminates the difficulties due to the singularity of the Green's function and improves the numerical efficiency. The sum of

[1] it can be of any shape

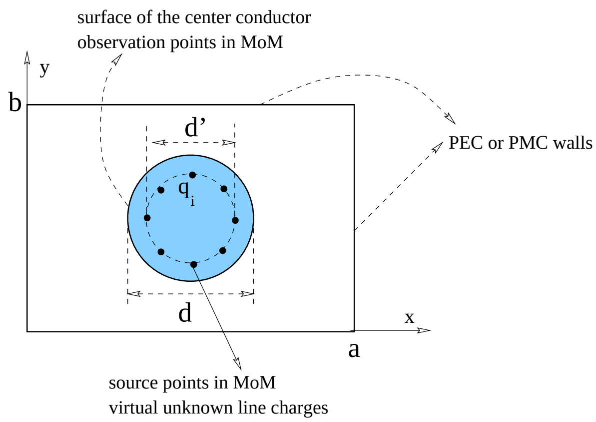


Fig. 2. Replacement of the inner conductor with line charges

the charges q_i represents the solution for the capacitance:

$$C = \frac{Q}{V} = \sum_{i=1}^{N} q_i \qquad (3)$$

To enhance the accuracy of the discretized boundary condition a sufficient number of observation points M ($M > N$) are considered and Eq. 2 is solved via a least square method. Using the appropriate Green's function the above procedure can be applied to all cases shown in Fig. 1 as well as trough and slab lines.

B. Green's Function

The method of images enables us to replace all the PEC and/or PMC walls with an infinite set of image charges with appropriate polarity. The static potential of the infinite periodic distribution of image charges can be expressed in terms of ϑ-functions as described in [11]. As an example, when all the side walls are made of perfect electric conductors, the Green's function is:

$$G(z, z') = Re\{W(z, z')\}$$

where $W(z, z')$ is given by:

$$W(z,z') = -\frac{1}{2\pi}\Big\{ln\,\vartheta_1\big(\frac{z-z'}{2a}\big) + ln\,\vartheta_1\big(\frac{z+z'}{2a}\big) - ln\,\vartheta_1\big(\frac{z-\bar{z}'}{2a}\big) - ln\,\vartheta_1\big(\frac{z+\bar{z}'}{2a}\big)\Big\} \qquad (4)$$

in which $z \triangleq x + i\,y$ is the observation point, $z' \triangleq x' + i\,y'$ represents the source point, and $\bar{z}$ means the complex conjugate. The ϑ-function is given by the following exponentially convergent series [11]:

$$\vartheta_1(z) = 2\sum_{m=0}^{\infty} q^{(m+\frac{1}{2})^2}\,(-1)^m\,sin[(2m+1)\pi\,z] \quad (5)$$

where $q = e^{-\pi\,b/a}$. a and b are the dimensions of the enclosure which are depicted in Fig. 2. As another example the Green's function for Fig. 1(a) is:

$$\begin{aligned} G(z,z') = -\frac{1}{2\pi}Re\Big\{ & ln\,\vartheta_1(\frac{z-z'}{4a}) + ln\,\vartheta_1(\frac{z+z'}{4a}) \\ & - ln\,\vartheta_1(\frac{z-\bar{z}'}{4a}) + ln\,\vartheta_1(\frac{z+\bar{z}'}{4a}) \\ & - ln\,\vartheta_1(\frac{z-z'-2a}{4a}) + ln\,\vartheta_1(\frac{z-\bar{z}'-2a}{4a}) \\ & - ln\,\vartheta_1(\frac{z+z'-2a}{4a}) + ln\,\vartheta_1(\frac{z+\bar{z}'-2a}{4a})\Big\} \end{aligned}$$

in which $q = e^{-\pi\,b/2a}$. Only the first 5 terms of Eq. 5 are usually sufficient to achieve an accuracy better than 10^{-5}. The electric field pattern of the TEM mode can also be obtained through the following equation:

$$\vec{E} = -\left(\frac{dW}{dz}\right)^{\star} \quad (6)$$

in which the differentiation can be performed analytically [11].

III. Numerical Results

A single computer subroutine was written to treat all possible combinations of PEC and PMC walls for a rectangular shield as well as trough and slab lines. Only circular inner conductor was considered which can be located anywhere within the shield. The numerical results are compared with those obtained from "Ansoft Maxwell 2D Parameter Extractor" which is specifically designed for 2D quasi-TEM problems.

The characteristic impedance of a rectangular coaxial line with a PEC shield was calculated for different values of the diameter of the inner conductor and excellent agreement with finite-element solution is observed as shown in Fig. 3.

As another example the even and odd mode characteristic impedances of a coupled slab line with a rectangular enclosure were evaluated. The geometry of the structure and its even and odd mode equivalents are shown in Fig. 4 and the corresponding impedances vs. the spacing between the two conductors are plotted in Fig. 5. Behavior of the even and odd mode impedances for the coupled slab line as a function of height of the enclosure is also shown in Fig. 6. In all different cases $60(=M)$ observation points were chosen to enforce the boundary condition on the center conductor and the number of unknowns was $N = 20$. The diameter of the fictitious surface charge d' was chosen to be $d' = 0.4d$.

It was found that as long as a sufficient number of observation points are chosen (typically $M > 50$) the method is quite robust against the number of unknowns and d'. For example, with $M = 60$ and

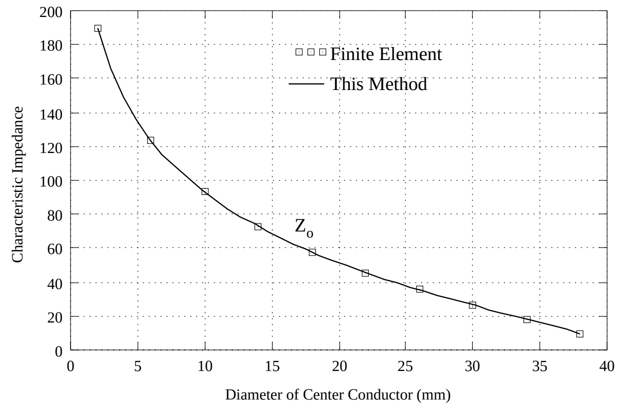
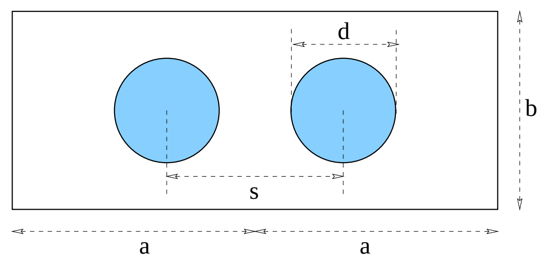
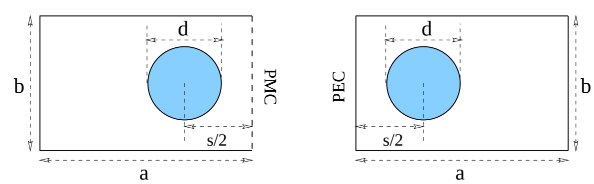


Fig. 3. Characteristic impedance of rectangular coaxial line. $a = 50mm$, $b = 40mm$



(a) Slab line



(b) Even Mode (c) Odd Mode

Fig. 4. Shielded coupled slab line

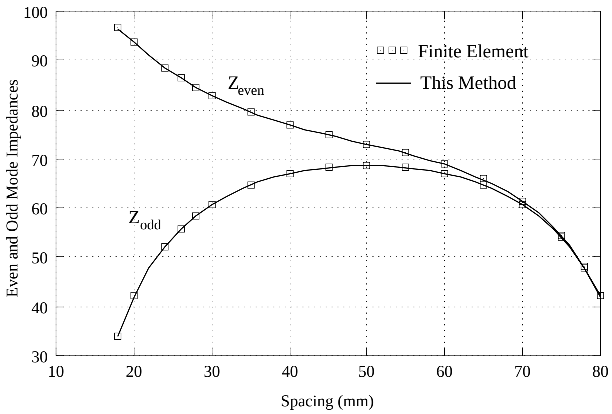


Fig. 5. Even and odd mode impedances for coupled slab line shown in Fig. 4. $a = 50mm$, $b = 40mm$, $d = 15mm$

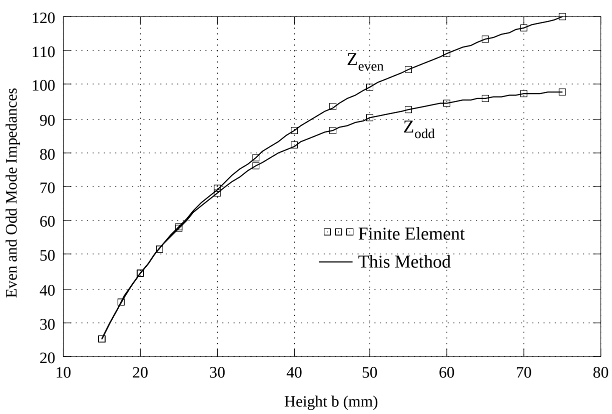


Fig. 6. Even and odd mode impedances for coupled slab line shown in Fig. 4. $a = 50mm$, $s = 50mm$, $d = 12mm$

$N = 20$ the largest deviation of the impedance values is less than $\%0.1$ when the diameter d' varies in a range of $0.2d < d' < 0.9d$. With $M = 60$ and $d' = 0.4d$ the maximum error is less than $\%0.15$ for $10 < N < 30$. In addition to robustness and accuracy the present method is quite fast which is mainly due to fast convergence of the Green's function and the least square solution of the matrix equation, *e.g.* generating the entire plot shown in Fig. 6 which is composed of 122 points for Z_{even} and Z_{odd} takes $14sec$ on a Pentium III 450MHz.

IV. Conclusions

A fast and accurate numerical method for evaluation of the TEM properties of homogeneous multiconductor transmission lines with rectangular enclosure and arbitrary combination of PEC and PMC walls was developed and verified. The characteristic impedances of a single rectangular-coaxial line as well as a coupled slab line were calculated and excellent agreement with those obtained from a finite-element solution was observed. Robustness, accuracy, and computational speed of the proposed approach were also addressed. This method can be efficiently incorporated into the design and optimization of combline and interdigital bandpass filters as well as multiple coupled coaxial cavity filters with a quasi-TEM approximation [9, 10].

Acknowledgments

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